Math 13: Multivariable Calculus

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Placement Test
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For this course, we will be using the textbook "Calculus: Early Transcendentals" by James Stewart, 8th edition. The course covers topics in multivariable calculus including partial derivatives, multiple integrals, and vector calculus.

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1 Iterated Integrals

1.1 Introduction to Integration, Iterated Integrals

Definition 1.1.1 (Double Integral)
The **double integral** of a function $f(x, y)$ over a rectangular region $R = [a, b] \times [c, d]$ is defined as:

$$\iint_R f(x, y)\, dA = \int_a^b \int_c^d f(x, y)\, dy\, dx$$

where $dA = dy\, dx$ represents the area element in the xy-plane.

Definition 1.1.2 (Triple Integral)
The **triple integral** of a function $f(x, y, z)$ over a rectangular region $R = [a, b] \times [c, d] \times [e, f]$ is defined as:

$$\iiint_R f(x, y, z)\, dV = \int_a^b \int_c^d \int_e^f f(x, y, z)\, dz\, dy\, dx$$

where $dV = dz\, dy\, dx$ represents the volume element in the xyz-space.

Definition 1.1.3 (Iterated Integral)
An **iterated integral** is an integral that is computed in steps, where the result of one integral is used as the integrand for the next. For example, a double integral can be expressed as:

$$\iint_R f(x, y)\, dA = \int_a^b \left(\int_c^d f(x, y)\, dy \right) dx$$

If $f(x, y)$ is continuous on R, then:

$$\iint_R f(x, y)\, dA = \lim_{\Delta x \to 0, \Delta y \to 0} \sum_{(x_i, y_j) \in R} f(x_i, y_j) \Delta A_{ij}$$

Proposition 1.1.4 (Linearity of Integrals)
For any constants a and b, and continuous functions $f(x, y)$ and $g(x, y)$ on region R:

$$\iint_R [af(x, y) + bg(x, y)]\, dA = a \iint_R f(x, y)\, dA + b \iint_R g(x, y)\, dA$$

Proposition 1.1.5 (Scaling of Integrals)
If c is a constant, then:

$$\iint_R cf(x, y)\, dA = c \iint_R f(x, y)\, dA$$

Proposition 1.1.6
If $g(x, y) \geq f(x, y)$ for all $(x, y) \in R$, then:

$$\iint_R g(x, y)\, dA \geq \iint_R f(x, y)\, dA$$

1.2 Fubini's Theorem, Integration over Non-Rectangular Regions

Theorem 1.2.1 (Fubini's Theorem)
If $f(x, y)$ is continuous on a rectangular region $R = [a, b] \times [c, d]$, then:

$$\iint_R f(x, y)\, dA = \int_a^b \left(\int_c^d f(x, y)\, dy \right) dx = \int_c^d \left(\int_a^b f(x, y)\, dx \right) dy$$

Proposition 1.2.2

$$\iint_R g(x)h(y)\, dA = \int_a^b g(x)\, dx \int_c^d h(y)\, dy$$

where $R = [a, b] \times [c, d]$ and $g(x)$ and $h(y)$ are continuous functions.

Proposition 1.2.3
If $m \leq f(x, y) \leq M$ for all $(x, y) \in R$, then:

$$m \cdot \text{Area}(R) \leq \iint_R f(x, y)\, dA \leq M \cdot \text{Area}(R)$$

where Area(R) is the area of region R.

1.3 Integration in Polar Coordinates

1.4 Applications of Double Integrals

1.5 Triple Integrals

1.6 Cylindrical Coordinates

1.7 Spherical Coordinates

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